Int. J. Heat Mass Transfer. Vol. 31, No. 8, pp. 1613–1617, 1988
Printed in Great Britain

0017–9310/88 \$3.00+0.00

A closed form solution for average heat transfer coefficient during forced vapour flow condensation on inclined surfaces

MAZHAR ÜNSAL†

Department of Mechanical Engineering, University of Hawaii at Manoa, Honolulu, Hawaii, U.S.A.

(*Received* 1 *May* 1987 *and in final form* 9 *December* 1987)

Abstract—Heat transfer during condensation of a flowing vapour on an isothermal plate of arbitrary inclination is investigated analytically. The analysis is based on a previously reported perturbation solution of the film stability problem. A closed form expression is obtained for the dimensionless average heat transfer coefficient which is compared with available solutions valid for the special cases of condensation on horizontal and vertical surfaces.

INTRODUCTION

HEAT TRANSFER during forced flow vapour condensation of a saturated vapour on isothermal surfaces has been the subject of numerous previous articles. To the best knowledge of the author, there is no prediction available in the literature regarding average heat transfer coefficients for forced vapour condensation on isothermal flat surfaces of arbitrary inclination. The present study is based on a perturbation analysis and leads to a closed form expression for the average heat transfer coefficient. Among the many previous works for the prediction of local heat transfer coefficients on a vertical surface are those reported in refs. [1–3]. Thermal convection terms in the energy equation and acceleration terms in the momentum equation of the condensate film were neglected in refs. [1, 2]. Denny and Mills [3] accounted for the convection and acceleration effects by means of a numerical analysis. They reported results in dimensional form and only for three different values of the vapour velocity. Result reported in all three studies are further restricted to local values of the heat transfer coefficient. The purpose of the present investigation is the presentation of a dimensionless closed form formula for the average heat transfer coefficient which may be readily used in many different practical situations, i.e. for many different values of the dimensionless parameters of the physical process.

ANALYSIS

A first-order perturbation solution for forced vapour flow condensation on inclined isothermal surfaces was previously reported in ref. [4]. The temperature distribution in the condensate film is given by ref. [4]

$$\theta = \theta_0 + \alpha\theta_1 + O(\alpha^2) \tag{1}$$

where

$$\theta_0 = y/\eta \tag{2}$$

and

$$\theta_1 = Pr\,R[-\eta^{-2}\eta_t(y^2-\eta^2)/6-(2C\eta^{-2}\eta_x+C_x\eta^{-1}) \times (y^3-\eta^3)/12+n\eta^{-2}\eta_x(y^4-\eta^4)/40]y. \tag{3}$$

Equation (1), being valid for a wavy condensate film, is also applicable for a steady condensate film. The steady-state condensate film thickness is governed by the following non-linear ordinary differential equation [4]:

$$k_0/\bar{\eta}-k_2\bar{\eta}^2\bar{\eta}_x-k_3U\bar{\eta}_x+O(\varepsilon^2) = \frac{\mathrm{d}}{\mathrm{d}x}[k_5\bar{\eta}^3\bar{\eta}_x+k_9\bar{\eta}^3\bar{\eta}_{xxx}+k_{10}\bar{\eta}_x+k_{13}U^2\bar{\eta}^2\bar{\eta}_x - k_{15}\bar{\eta}^6\bar{\eta}_x-k_{16}U\bar{\eta}^4\bar{\eta}_x]+O(\alpha^3). \tag{4}$$

Coefficients k_0–k_{16} were reported in ref. [4]. Terms of $O(\varepsilon^2)$ in equation (4) represent the effect of finite amplitude wave motion on the condensate film and this term will vanish for a steady film. Additionally, terms multiplied by negligibly small coefficients k_5, k_9 and k_{10} will be neglected. The characteristic length in equation (4) is the wavelength, λ, which does not fit in with the physics of a steady condensate film. The following transformations will be introduced into equation (4) in order to change the characteristic dimension from wavelength to $(2\nu^2/g)^{1/3}$ which is the proper characteristic dimension for a steady film

$$\Delta = k_2\bar{\eta}^2/Uk_3 \tag{5}$$

$$\xi = 4k_0k_2x/U^2k_3^2. \tag{6}$$

Substitution of equations (5) and (6) into equation (4) and taking $\varepsilon = k_5 = k_9 = k_{10} = 0$ in equation (4) yields

$$1-2\Delta\Delta_\xi-2\Delta = F[-s_1(5\Delta^2\Delta_\xi^2+2\Delta^3\Delta_{\xi\xi}) + s_2(\Delta\Delta_{\xi\xi}+\Delta_\xi^2/2)+s_3(3\Delta\Delta_\xi^2+2\Delta^2\Delta_{\xi\xi})]+O(F^2) \tag{7}$$

† On leave from Department of Mechanical Engineering, University of Gaziantep, 27310 Gaziantep, Turkey.

NOMENCLATURE

c_p liquid specific heat
F acceleration effect parameter, $c_p\Delta T/h_{fg}\,Pr$
$F\,Pr$ heat capacity parameter, $c_p\Delta T/h_{fg}$
Fr Froude number, u^2/nL
g gravitational acceleration
h heat transfer coefficient
$\bar{h}$ average heat transfer coefficient
h_{fg} enthalpy of phase change
k liquid conductivity
$\tilde{L}$ length of plate
L $\tilde{L}/(2v^2/g)^{1/3}$
M μ_v/μ
n $2\cos\phi$
Pr Prandtl number
R $g\tilde{\eta}_0^3/2v^2$
$\tilde{t}$, t time, dimensionless time
$\tilde{T}$, $\tilde{T}_s$, $\tilde{T}_w$ liquid, vapour, interface temperature
ΔT $\tilde{T}_s-\tilde{T}_w$
u $\tilde{U}_{v0}/(gv/2)^{1/3}$
U_0 characteristic velocity, $g\tilde{\eta}_0^2/2v$
$\tilde{U}_{v0}$ vapour $\tilde{x}$-component velocity
U $\tilde{U}_{v0}/\tilde{U}_0$
$(\tilde{x}, \tilde{y})$, (x, y) dimensional, dimensionless, Cartesian coordinates.

Greek symbols
α wave number, $2\pi\tilde{\eta}_0/\lambda$
γ ρ_v/ρ
Δ defined in equation (5)
Δ_{0L} defined in equation (25)
ε small parameter
$\tilde{\eta}$ film thickness
$\tilde{\eta}_0$ local film thickness at downstream end of plate
$\bar{\eta}$ dimensionless steady flow film thickness
η dimensionless unsteady film thickness
θ dimensionless temperature
λ wavelength
μ, μ_v liquid, vapour viscosity
v liquid kinematic viscosity
ξ defined in equation (6)
ξ_L defined in equation (24)
ρ, ρ_v liquid, vapour density
ϕ angle of plate with vertical.

Subscripts
ξ, $\tilde{y}$, x partial differentiation with respect to the subscript.

Dimensionless terms
θ $(\tilde{T}-\tilde{T}_w)/(\tilde{T}_s-\tilde{T}_w)$
y $\tilde{y}/\tilde{\eta}_0$
η $\tilde{\eta}/\tilde{\eta}_0$
t $\alpha\tilde{U}_0\tilde{t}/\tilde{\eta}_0$
x $\alpha\tilde{x}/\tilde{\eta}_0$.

where

$$s_1 = 4k_{15}/Rk_2^2 \tag{8}$$

$$s_2 = 8k_{13}/Rk_3 \tag{9}$$

$$s_3 = -4k_{16}/Rk_2k_3. \tag{10}$$

The following expansion will be utilized to find a first-order asymptotic solution of equation (7):

$$\Delta = \Delta_0+F\Delta_1+O(F^2) \tag{11}$$

subject to the initial condition

$$\Delta(0) = 0. \tag{12}$$

Substituting equation (11) into equations (7) and (12), and equating like powers of F, one obtains the zeroth-order and the first-order problems which may be solved in succession to yield

$$\Delta_0 = -1+(1+\xi)^{1/2} \tag{13}$$

$$\Delta_1 = s_4/(1+\xi)-s_5(1+\xi)^{-1/2}\ln(1+\xi) -(s_1+s_3/4)+(3s_1\xi/8-s_4+s_1+s_3/4)\times(1+\xi)^{-1/2} \tag{14}$$

where

$$s_4 = (-s_1+s_2/2-s_3)/2 \tag{15}$$

$$s_5 = (s_1+s_3-s_2/2)/8. \tag{16}$$

The heat transfer coefficient is determined from

$$h = -k\tilde{T}_{\tilde{y}}/(\tilde{T}_w-\tilde{T}_s) \quad \text{at}\ \tilde{y}=0 \tag{17}$$

and its average value from

$$\bar{h} = \frac{1}{\tilde{L}}\int_0^{\tilde{L}} h\,\mathrm{d}\tilde{x}. \tag{18}$$

Equations (1)–(3) with equations (5), (6) and (17) result in the following expression for the dimensionless local heat transfer coefficient:

$$(h/k)(2v^2/g)^{1/3}(k_3u/k_2)^{1/2} = \Delta^{-1/2}+F(2b_1(\alpha/k_2)\Delta^{1/2}\Delta_\xi +2b_2(\alpha/k_3)\Delta^{-1/2}\Delta_\xi)+O(F^2) \tag{19}$$

where

$$b_1 = n(8+3F)Pr/80(1+F) \tag{20}$$

$$b_2 = F\,Pr/24(1+F). \tag{21}$$

Finally, from equations (19) and (18), the dimensionless average heat transfer coefficient is obtained as

$$
\begin{aligned}
&(3\bar{h}/4k)(2\nu^2/g)^{1/3}(4FL/n)^{1/4} \\
&= (k_2/\alpha n)^{1/4}\xi_L^{-3/4}\Delta_{0L}^{1/2}\{\Delta_{0L}+3 \\
&\quad +F[(\alpha b_1/k_2-3s_1/16)\Delta_{0L} \\
&\quad +3(s_1/8+s_3/8+\alpha b_2/k_3) \\
&\quad +(3s_4/2+6s_5)\Delta_{0L}^{-1/2}\tan^{-1}(\Delta_{0L})^{1/2} \\
&\quad -3s_5\Delta_{0L}^{-1}\ln(1+\Delta_{0L})]+O(F^2)\}
\end{aligned} \tag{22}
$$

where

$$L = \tilde{L}/(2\nu^2/g)^{1/3} \tag{23}$$

$$\xi_L = 4\alpha F k_2 L/(k_3 u)^2 \tag{24}$$

$$\Delta_{0L} = -1+(1+\xi_L)^{1/2}. \tag{25}$$

Equation (22) is valid for a plate of arbitrary inclination. For a horizontal surface ($\phi = 90°$, $-90°$), $n = 2\cos\phi$ vanishes and equation (22) simplifies to

$$
\begin{aligned}
&(\bar{h}/2k)(\tilde{L}\nu/\tilde{U}_{v0})^{1/2} = (k_3/2\alpha F)^{1/2} \\
&\quad \times[1+F(\alpha b_2/k_3+s_2/16)+O(F^2)].
\end{aligned} \tag{26}
$$

For condensation of a quiescent vapour ($\tilde{U}_{v0} = 0$) on an inclined surface, equation (22) simplifies to

$$
\begin{aligned}
&(3\bar{h}/4k)(2\nu^2/k)^{1/3}(4FL/n)^{1/4} = (k_2/\alpha n)^{1/4} \\
&\quad \times[1+F(\alpha b_1/k_2-3s_1/16)+O(F^2)].
\end{aligned} \tag{27}
$$

DISCUSSION

Equation (22) gives the dimensionless average heat transfer coefficient for forced vapour flow condensation on an isothermal inclined surface and is depicted in Figs. 1–5 for five different Prandtl numbers. The lowest curves in Figs. 1–5 corresponding to $Fr = 0$ is for condensation of a quiescent vapour. Numerical solution of the full boundary layer equations for $Fr = 0$ was reported in ref. [5].

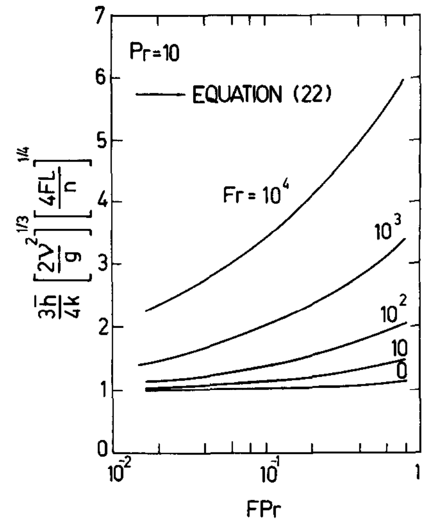

FIG. 2. Dimensionless average heat transfer coefficient for a vertical surface ($n = 2$) when $Pr = 10$.

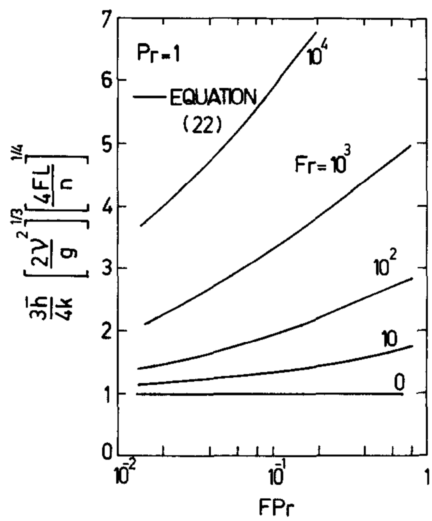

FIG. 3. Dimensionless average heat transfer coefficient for a vertical surface ($n = 2$) when $Pr = 1$.

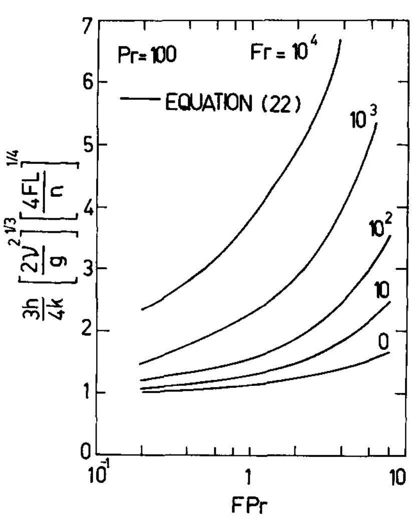

FIG. 1. Dimensionless average heat transfer coefficient for a vertical surface ($n = 2$) when $Pr = 100$.

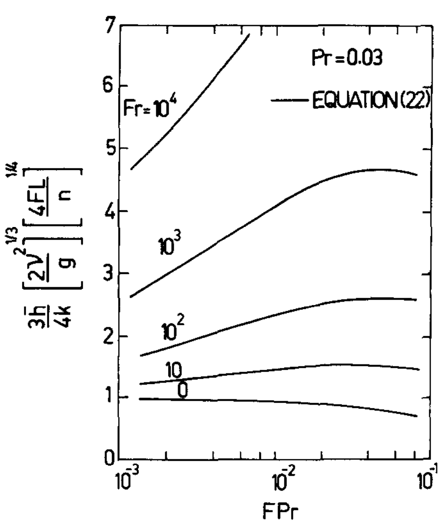

FIG. 4. Dimensionless average heat transfer coefficient for a vertical surface ($n = 2$) when $Pr = 0.03$.

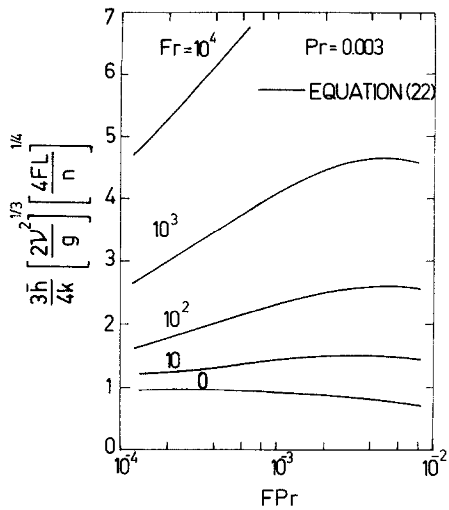

FIG. 5. Dimensionless average heat transfer coefficient for a vertical surface (n = 2) when Pr = 0.003.

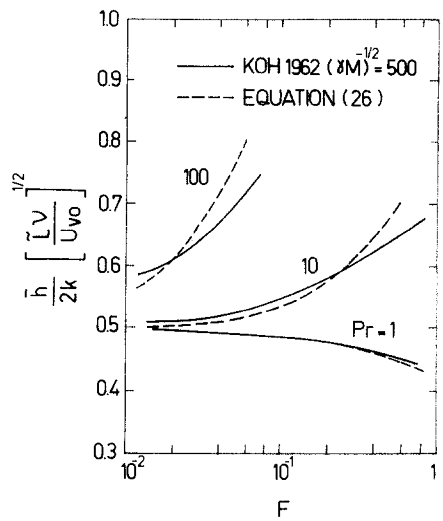

FIG. 7. Comparison of equation (26) with the results of Koh [6] (n = 0, ϕ = 90°).

Figures 2 and 3 of ref. [5] for Pr = 100, 10, 1, 0.03, and 0.003 are in complete agreement with the curves corresponding to $Fr = 0$ in Figs. 1–5 of the present study. This validates the applicability of equation (22) in the small Froude number limit. The present analysis which is a follow up to the analysis reported in ref. [4] is based on a generalized version of the 'asymptotic shear stress' interfacial boundary condition (equation (28) of ref. [4]). It is concluded, therefore, that the 'asymptotic shear stress' interfacial condition is a valid approximation at small Froude numbers. Equation (22) being valid for small Froude numbers should be expected to be valid for all Froude numbers if its validity can be demonstrated for $Fr = \infty$. In this limit, equation (22) simplifies into equation (26) which is also valid for forced vapour flow condensation on a horizontal surface. This problem has been previously solved by Koh [6] considering the full vapour thermal boundary layer equation. Equation (26) is compared with the results of Koh [6] in Figs. 6–8. It is seen from these figures that agreement with Koh's results are good at small Prandtl numbers and worse at large Prandtl numbers. It is thus concluded that equation (26) is valid at the asymptotic limit when $\gamma M = 0$. It has been previously mentioned in ref. [7] that analyses of forced vapour flow condensation based on the 'asymptotic shear stress' boundary condition is generally valid for $(\gamma M)^{1/2}/F < 2$ and the present results are in agreement with this observation. The present analysis is a first-order perturbation solution with respect to the acceleration effect parameter, F, and applicability of equations (22), (26), and (27) is further limited to values of $F < 1$.

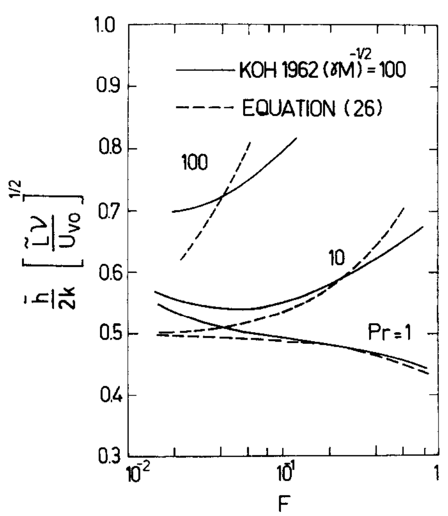

FIG. 6. Comparison of equation (26) with the results of Koh [6] (n = 0, ϕ = 90°).

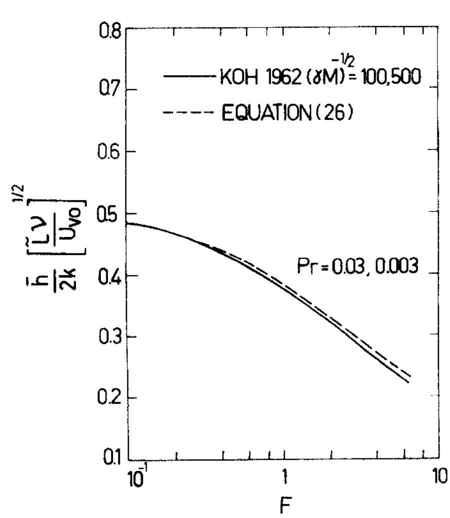

FIG. 8. Comparison of equation (26) with the results of Koh [6] (n = 0, ϕ = 90°).

REFERENCES

1. H. R. Jacobs, An integral treatment of combined body force and forced convection in laminar film condensation, *Int. J. Heat Mass Transfer* **9**, 637–648 (1966).
2. I. G. Shekriladze and V. I. Gomelauri, Theoretical study of laminar film condensation of flowing vapour, *Int. J. Heat Mass Transfer* **9**, 581–591 (1966).
3. V. E. Denny and A. F. Mills, Nonsimilar solutions for laminar film condensation on a vertical surface, *Int. J. Heat Mass Transfer* **12**, 965–979 (1969).
4. M. Ünsal, A linearized stability analysis of forced vapor flow condensation, *Proceedings of the NATO–ARW on Advances in Two-phase Flows and Heat Transfer*, F.R.G., 31 August–3 September (1982).
5. J. C. Y. Koh, E. M. Sparrow and J. P. Hartnett, The two phase boundary layer in laminar film condensation, *Int. J. Heat Mass Transfer* **2**, 69–82 (1961).
6. J. C. Y. Koh, Film condensation in a forced convection boundary layer flow, *Int. J. Heat Mass Transfer* **5**, 941–954 (1962).
7. D. H. Cho and M. Epstein, Laminar film condensation of flowing vapor on a horizontal melting surface, *Int. J. Heat Mass Transfer* **20**, 23–30 (1977).

UNE SOLUTION ANALYTIQUE DU COEFFICIENT DE TRANSFERT MOYEN DE CHALEUR POUR L'ECOULEMENT FORCE DE VAPEUR AVEC CONDENSATION SUR DES SURFACES INCLINEES

Résumé—On étudie analytiquement le transfert thermique pendant la condensation d'une vapeur s'écoulant sur une plaque isotherme inclinée. L'analyse est basée sur une solution de perturbation déjà publiée pour un problème de stabilité de film. Une expression est obtenue pour le coefficient sans dimension de transfert de chaleur moyen et il est comparé avec d'autres solutions valables pour les cas particuliers de condensation sur des surfaces horizontales ou verticales.

EINE GESCHLOSSENE LÖSUNG FÜR DEN MITTLEREN WÄRMEÜBERGANGSKOEFFIZIENTEN BEI DER KONDENSATION EINER ERZWUNGENEN DAMPFSTRÖMUNG AN GENEIGTEN OBERFLÄCHEN

Zusammenfassung—Betrachtet wird der Wärmetransport während der Kondensation eines strömenden Dampfes an einer isothermen Platte mit beliebigem Neigungswinkel. Die Untersuchung basiert dabei auf einer bereits vorgestellten Lösung des Filmstabilitätsproblems. Für den dimensionslosen mittleren Wärmeübergangskoeffizienten wird eine Gleichung in geschlossener Form vorgestellt, welche mit den bekannten Lösungen für die Kondensation an horizontalen und vertikalen Flächen verglichen wird.

ЗАМКНУТОЕ РЕШЕНИЕ ДЛЯ СРЕДНЕГО КОЭФФИЦИЕНТА ТЕПЛОПЕРЕНОСА ПРИ ВЫНУЖДЕННОЙ КОНДЕНСАЦИИ ПОТОКА ПАРА НА НАКЛОННЫХ ПОВЕРХНОСТЯХ

Аннотация—Аналитически исследуется теплоперенос при конденсации потока пара на изотермической пластине с произвольным углом наклона. Анализ основывается на полученном ранее решении методом возмущения задачи устойчивости пленки. Решение представлено в замкнутом виде для безразмерного среднего коэффициента теплопереноса и дано его сравнение с имеющимися решениями, которые являются справедливыми в особых случаях конденсации на горизонтальных и вертикальных поверхностях.